International Journal of Engineering Research in Current Trends (IJERCT)
ISSN: 2582-5488, Volume-1 Issue-1, December 2019

Some Fixed Point Theorem in Menger Space with Contractive Conditions of Occasionally Weakly Compatible (OWC) Mapping Via Integral Inequalities

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Abstract: The purpose of this paper is to establish some new fixed Point theorem in Menger space with various contractive conditions of occasionally weakly compatible mapping via integral inequalities.

Keywords: Menger space, Common fixed Point, Occasionally weakly Compatible mapping, t- norm.

1. INTRODUCTION

In 1942,the concept of probabilistic metric space was introduced by Menger[18] in Menger theory and solve when we don't know exactly the distance between two points but we know the probabilities of possible value of this distance and explain how to replace the numerical distance between two points. The 1980s was a period of great activity in fixed point theory, including many important results. Jangck[6,8] introduced the concept of compatible mapping and proved many theorems in metric space., Later on Singh and Jain [13] introduced the notion of weakly compatible maps in the setting of Menger space and prove some fixed point theorems. Later on, Pant[19] introduced the concept of reciprocally continuous mappings and proved some common fixed point theorems. Rohan et al. [20] introduced the concept Common fixed point of of compatible mapping of type (C) in 2004.Singh et al. [21] introduced the concept of compatible mapping of type (E) in 2007, Afterword Thagafiand Shahzad[17] introduced the concept of occasionally weakly compatible mappings. Abbas And Rhoades [1,2,3] used Common fixed point theorems for hybrid pairs of occasionally weakly compatible mappings and proved some generalized contractive condition . Singh Ruchi et al.[22] used fixed point results in fuzzy Menger space with common property(E.A).Many authors have proved common fixed point theorems in Menger space for different contractive conditions. In this paper is to generalize some mixed type of contractive conditions, satisfying general contractive mappings such as Kannan type [10].we also present integral type common fixed point in Menger space. Our results improve, generalized and extend the results of jatin s.patel[15] by using Occasionally weakly compatible mappings.

2. PRELIMINARIES

In this section, we give some basic definitions which are useful for main result in this paper.

Definition 2.1.([19]) A mapping $T: [0,1] \times [0,1] \rightarrow [0,1]$ is called Triangular norm (or t-norm) if it satisfies the following properties:

(i) $T(0,0) = 0$ *and* $T(a,1) = a \quad \forall\, a \in [0,1]$

(ii) $T(a,b) = T(a,b) \;\forall\, a,b \in T(a,b) = T(a,b)\; [0,1]$

(iii) $T(a,b) \leq T(c,d)$ *whenever* $a \leq c$ *and* $b \leq d \quad \forall\, a,b,c,d \in [0,1]$

(iv) $T(T(a,b),c) = T\big(a,T(b,c)\big) \quad \forall\, a,b,c,d \in [0,1]$

Definition 2.2.([19]) A Menger Space is a Triplet (X,F,T) Where X is z non-empty Space and F is a Function defined on $X \times X$ to the set of distribution functions and T is t-norm such that following properties are satisfied :

(i) $F_{x,y}(0) = 0 \;\forall\; x, y \in X$

(ii) $F_{x,y}(t) = t \;\forall\; t > 0$ if and only if $x = y$

(iii) $F_{x,y}(t) = F_{y,x}(t) \;\forall\; x, y \in X$

(iv) $F_{x,y}(t) = 1$ and $F_{y,z}(t) = 1$ then $F_{x,z}(t+s) = 1 \;\forall\; x, y, z \in X$

Definition 2.3.([19]) A Menger Space is a Triplet (X,F,T) ,Where (X,F) is a Probabilistic Metric Space (PM-Space) and T is a t-norm such that for all x,y , $z \in X$ and all $t, s \geq 0$

$$F_{x,y}(s+t) \geq T(F_{x,z}(s), F_{z,y}(t))$$

Published By:
International Journal of Engineering Research in Current Trends

Definition 2.4([1]) Let f and g be a mapping from a Menger space (X, F, T) into itself. Then the mapping are said to be Compatible if for all t > 0, $\lim_{n\to\infty} F_{fgx_n, gfx_n}(t) = 1$

Whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x \in X$

Definition 2.5([7]) Let (X, F, T) be a Menger space, Where T is a continuous t-norm then

(i) A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if

$\lim_{n\to\infty} F x_n, x\ (t) = 1,\ \forall t > 0$

(ii) A sequence $\{x_n\}$ in X is said to be a Cauchy Sequence if for each $0 < \alpha < 1$ and t > 0 there exist a positive inter n_0 such that $Fx_n, x_m \geq 1 - \alpha$ For each $n,m \geq n_0$

(iii) A Menger space *(X,F,T)* is said to be Complete if every Cauchy Sequence in X converges to a point of it.

Definition 2.6([8] Let X be a set, f and g be a self maps of X then a point $x \in X$ is called a coincidence point of f and g if and only if fx = gx. We shall call w = fx =gx a point of coincidence of f and g.

Definition 2.7([7]) Two maps P and Q are said to weakly compatible if they commute at their coincidence points i.e if Pz = Qz some $z \in X$ then PQz = QPz

Example 2.1[1]] Let R be the usual metric space . P,Q,R→R by P(x) = 3x and Q(x) = x^2 for all $x \in R$ .Then P(x) = Q(x) for x = 0, 3 but PQ(0) = QP(0) and PQ(3) ≠ QP(3). P and Q are occasionally weakly compatible self maps but not weakly compatible.

Definition 2.9([8]) Let X be a set, f and g be a occasionally weakly compatible self maps of X. if f and g have a unique of coincidence i.e ,w = fx = gx then w is the unique common fixed point of f and g.

Lemma 2.1 ([14]) Let (*X*, *F*, *T*) be a Menger space and x,y ∈X. if there exists a constant h∈ (0,1) such that $F_{x,y}(ht) \geq F_{x,y}(t)$ for all t > 0 then x = y

Lemma 2.2([11]) Let X be a set, f and g be a occasionally weakly compatible self maps of X. if f and g have a unique point of coincidence i.e ,w = fx = gx then w is the unique common fixed point of f and g.

3. MAIN RESULT

Theorem (3.1): Let p,q,f and g be a self-maps of Menger space (X, F, T) .Let the pairs (p,f) and (q,g) be occasionally weakly compatible . if there exist h∈ (0,1) such that

$$F_{px,qy}(ht) \geq \min\{F_{fx,gy}(t), F_{fx,px}(t), \frac{\alpha F_{px,gy}(t) + \beta F_{qy,fx}(t),}{\alpha+\beta}, F_{fx,qy}(t), F_{qy,gy}(t)\}$$

For all x,y ∈X, and 0 < t < 1 then there exist a unique common fixed point of p,q,f and g.

Proof: Since the pairs (p,f) and (q,g) be owc. So there are points $x,y \in X$ such that px = fx and qy = gy .Now we shall show that px = qy, . if not then by inequality (3.1) we have

$$F_{px,qy}(ht) \geq \min\{F_{fx,gy}(t), F_{fx,px}(t), \frac{\alpha F_{px,gy}(t) + \beta F_{qy,fx}(t),}{\alpha+\beta}, F_{fx,qy}(t), F_{qy,gy}(t)\}$$

$$F_{px,qy}(ht) \geq \min\{F_{px,qy}(t), F_{px,px}(t), \frac{\alpha F_{px,qy}(t) + \beta F_{qy,px}(t),}{\alpha+\beta}, F_{px,qy}(t), F_{qy,qy}(t)\}$$

$$F_{px,qy}(ht) \geq \min\{F_{px,qy}(t),\ 1, F_{px,qy}(t)\ , F_{px,qy}(t), 1\}$$

$$F_{px,qy}(ht) \geq F_{px,qy}(t)$$

Therefore by lemma 2.1, we have px = qy i.e px = fx = qy = gy. Suppose that there is another point z such that pz = fz then by inequality (3.1) we have fz = qy = gy= z, So px = pz and w = px = fx is the unique point of coincidence of p and f . Similarly there is a unique point $z \in X$ such that z = qz = gz . Suppose that $w \neq z$ then by inequality (3.1)

$$F_{px,qy}(ht) \geq \min\{F_{fx,gy}(t), F_{fx,px}(t), \frac{\alpha F_{px,gy}(t) + \beta F_{qy,fx}(t),}{\alpha+\beta}, F_{fx,qy}(t), F_{qy,gy}(t)\}$$

$$F_{pw,qz}(ht) \geq \min\{F_{fw,gz}(t), F_{fw,pw}(t), \frac{\alpha F_{pw,gz}(t) + \beta F_{qz,fw}(t),}{\alpha+\beta}, F_{fw,qz}(t), F_{qz,qz}(t)\}$$

$$F_{w,z}(ht) \geq \min\{F_{w,z}(t), F_{w,w}(t), \frac{\alpha F_{w,z}(t)+\beta F_{z,w}(t),}{\alpha+\beta}, F_{w,z}(t), F_{z,z}(t)\}$$

$$F_{w,z}(ht) \geq \min\{F_{w,z}(t), 1, F_{w,z}(t), F_{w,z}(t), 1\}$$

$$F_{w,z}(ht) \geq F_{w,z}(t)$$

Therefore we have w = z then by lemma 2.2, z is a common fixed point of p,q,f and g.

Uniqueness: let z' be the another common fixed point of p,q,f and g then by inequality (3.1)

$$F_{pz,qz'}(ht) \geq \min\{F_{fz,gz'}(t), F_{fz,pz}(t), \frac{\alpha F_{pz,gz'}(t)+\beta F_{qz',fz}(t),}{\alpha+\beta}, F_{fz,qz'}(t), F_{qz',gz'}(t)\}$$

$$F_{z,z'}(ht) \geq \min\{F_{z,z'}(t), F_{z,z}(t), \frac{\alpha F_{z,z'}(t)+\beta F_{z',z}(t),}{\alpha+\beta}, F_{z,z'}(t), F_{z',z'}(t)\}$$

$$F_{z,z'}(ht) \geq \min\{F_{z,z'}(t), 1, F_{z,z'}(t), F_{z,z'}(t), 1\}$$

$$F_{z,z'}(ht) \geq F_{z,z'}(t)$$

By lemma 2.1, we have z=z'. Therefore z=z' is a unique common fixed point of p,q,f and g.

Theorem (3.2): Let p,q,f and g be a self-maps of Menger space (X, F, T) .Let the pairs (p,f) and (q,g) be occasionally weakly compatible . if there exist h∈ (0,1) such that

$$F_{px,qy}(ht) \geq \emptyset\left(\min\{F_{fx,gy}(t), F_{fx,px}(t), \frac{\alpha F_{px,gy}(t)+\beta F_{qy,fx}(t),}{\alpha+\beta}, F_{fx,qy}(t), F_{qy,gy}(t)\}\right)$$

For all x,y ∈X, and $[0,1]^5 \rightarrow [0,1]$ such that $\emptyset(t) > t$ and $0 < t < 1$ then there exist a unique common fixed point of p,q,f and g.

Proof: Since the pairs (p,f) and (q,g) be owc.So there are points x,y ∈X such that px = fx and qy = gy .Now we shall show that px = qy. if not then by inequality (3.1) we have

$$F_{px,qy}(ht) \geq \emptyset\left(\min\{F_{fx,gy}(t), F_{fx,px}(t), \frac{\alpha F_{px,gy}(t)+\beta F_{qy,fx}(t),}{\alpha+\beta}, F_{fx,qy}(t), F_{qy,gy}(t)\}\right)$$

$$F_{px,qy}(ht) \geq \emptyset\left(\min\{F_{px,qy}(t), F_{px,px}(t), \frac{\alpha F_{px,qy}(t)+\beta F_{qy,px}(t),}{\alpha+\beta}, F_{px,qy}(t), F_{qy,qy}(t)\}\right)$$

$$F_{px,qy}(ht) \geq \emptyset\left(\min\{F_{px,qy}(t), 1, F_{px,qy}(t), F_{px,qy}(t), 1\}\right)$$

$$F_{px,qy}(ht) \geq \emptyset(F_{px,qy}(t))$$

$$F_{px,qy}(ht) \geq F_{px,qy}(t)$$

Therefore by lemma 2.1, we have px = qy, i.e px = fx = qy = gy. Suppose that there is another point z such that pz =fz then by inequality (3.1) we have pz = fz = qy = gy So px = pz and w =px =fx is the unique point of coincidence of p and f .Similarly there is a unique point z ∈ X such that z=qz=gz Suppose that w ≠ z then by

$$F_{pw,qz}(ht) \geq \emptyset\left(\min\{F_{fw,gz}(t), F_{fw,pw}(t), \frac{\alpha F_{pw,gz}(t)+\beta F_{qz,fw}(t),}{\alpha+\beta}, F_{fw,qz}(t), F_{qz,gz}(t)\}\right)$$

$$F_{w,z}(ht) \geq \emptyset\left(\min\{F_{w,z}(t), F_{w,w}(t), \frac{\alpha F_{w,z}(t)+\beta F_{z,w}(t),}{\alpha+\beta}, F_{w,z}(t), F_{z,z}(t)\}\right)$$

$$F_{w,z}(ht) \geq \emptyset\left(\min\{F_{w,z}(t), 1, F_{w,z}(t), F_{w,z}(t), 1\}\right)$$

$$F_{w,z}(ht) \geq \emptyset(F_{w,z}(t))$$

$$F_{w,z}(ht) \geq F_{w,z}(t)$$

Therefore we have w = z then by lemma 2.1 , z is a common fixed point of p,q,f and g.

Uniqueness: let z' be the another common fixed point of p,q,f and g then by inequality (3.1)

$$F_{pz,qz'}(ht) \geq \emptyset(\min\{F_{fz,gz'}(t), F_{fz,pz}(t), \frac{\alpha F_{pz,gz'}(t)+\beta F_{qz',fz}(t)}{\alpha+\beta}, F_{fz,qz'}(t), F_{qz',gz'}(t)\})$$

$$F_{z,z'}(ht) \geq \emptyset(\min\{F_{z,z'}(t), F_{z,z}(t), \frac{\alpha F_{z,z'}(t)+\beta F_{z',z}(t)}{\alpha+\beta}, F_{z,z'}(t), F_{z',z'}(t)\})$$

$$F_{z,z'}(ht) \geq \emptyset(\min\{F_{z,z'}(t), 1, F_{z,z'}(t), F_{z,z'}(t), 1\})$$

$$F_{z,z'}(ht) \geq \emptyset(F_{z,z'}(t))$$

$$F_{z,z'}(ht) \geq F_{z,z'}(t)$$

By lemma 2.1, we have z=z' . Therefore z=z' is a unique common fixed point of p,q,f and g.

Theorem (3.3): Let p,q,f and g be a self-maps of Menger space (X, F, T) .Let the pairs (p,f) and (q,g) be occasionally weakly compatible . if there exist h∈ (0,1) such that

$$F_{px,qy}(ht) \geq \emptyset(\min\{F_{fx,px}(t), F_{fx,gy}(t), F_{qy,fx}(t), \frac{\alpha F_{px,gy}(t)+\beta F_{qy,fx}(t)}{\alpha+\beta}, F_{px,gy}(t), F_{qx,qy}(t)\})$$

For all x,y ∈X, and $[0,1]^6 \rightarrow [0,1]$ such that $\emptyset(1, t, t, , t, t, 1) > t$ and $0 < t < 1$ then there exist a unique common fixed point of p,q,f and g.

Proof: Same proof.

4. RELATED RESULTS FOR CONTRACTIVE CONDITION OF INTEGRAL TYPES.

Lemma (4.1): Let p,q,f and g be a self-maps of Menger space (X, F, T) .Let the pairs (p,f) and (q,g) be occasionally weakly compatible . if there exist h∈ (0,1) such that

$$\int_0^{F_{x,y}(ht)} \varphi(t)dt \geq \int_0^{F_{x,y}(t)} \varphi(t)dt$$

Where $\varphi[0,1] \rightarrow [0,1]$ is a summable non-negative Integrable function such that
$\int_0^1 \varphi(t)dt > 0$ then x = y

Proof: Since $\int_0^{F_{x,y}(ht)} \varphi(t)dt \geq \int_0^{F_{x,y}(t)} \varphi(t)dt$

$$\Rightarrow \int_0^{F_{x,y}(t)} \varphi(t)dt \geq \int_0^{F_{x,y}(h^{-1}t)} \varphi(t)dt$$

Similarly ,we can inductively write for n$\in N$

$$\Rightarrow \int_0^{F_{x,y}(t)} \varphi(t)dt \geq \int_0^{F_{x,y}(h^{-2}t)} \varphi(t)dt$$

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$$\Rightarrow \int_0^{F_{x,y}(t)} \varphi(t)dt \geq \int_0^{F_{x,y}(h^{-n}t)} \varphi(t)dt \rightarrow \int_0^1 \varphi(t)dt$$

As n$\rightarrow \infty$

$$\Rightarrow \int_0^{F_{x,y}(t)} \varphi(t)dt - \int_0^1 \varphi(t)dt \geq 0$$

$$\Rightarrow \int_0^{F_{x,y}(t)} \varphi(t)dt - \left(\int_0^{F_{x,y}(t)} \varphi(t)dt + \int_{F_{x,y}(t)}^1 \varphi(t)dt\right) \geq 0$$

$$\Rightarrow \int_{F_{x,y}(t)}^1 \varphi(t)dt) \leq 0$$

For all t > 0 , $F_{x,y}$ (t) ≥ 0 then x = y

Remark: by setting $\varphi(t) = 1$ for each t ≥ 0 then

$$\int_0^{F_{x,y}(\mathrm{ht})} dt = [t]_0^{F_{x,y}(\mathrm{ht})} = F_{x,y}(\mathrm{ht}) \geq F_{x,y}(\mathrm{t}) = \int_0^{F_{x,y}(\mathrm{t})} dt$$

Theorem (4.1): Let p,q,f and g be a self-maps of Menger space (X, F, T) .Let the pairs (p,f) and (q,g) be occasionally weakly compatible . if there exist h∈ (0,1) such that

$$\int_0^{F_{px,qy}(\mathrm{ht})} \varphi(t)dt \geq \int_0^{\min\{F_{fx,gy}(\mathrm{t}), F_{fx,px}(\mathrm{t}), \frac{\alpha F_{px,gy}(\mathrm{t}) + \beta F_{qy,fx}(\mathrm{t}),}{\alpha+\beta}, F_{fx,qy}(\mathrm{t}), F_{qy,gy}(\mathrm{t})\})} \varphi(t)dt$$

For all x,y ∈X, and 0 < t < 1 then there exist a unique common fixed point of p,q,f and g.

Proof: Since the pairs (p,f) and (q,g) be owc. So there are points x,y ∈ X such that px = fx and qy = gy .Now we shall show that px = qy, . if not then by inequality (4.1) we have

$$\int_0^{F_{px,qy}(\mathrm{ht})} \varphi(t)dt \geq \int_0^{\min\{F_{fx,gy}(\mathrm{t}), F_{fx,px}(\mathrm{t}), \frac{\alpha F_{px,gy}(\mathrm{t}) + \beta F_{qy,fx}(\mathrm{t}),}{\alpha+\beta}, F_{fx,qy}(\mathrm{t}), F_{qy,gy}(\mathrm{t})\})} \varphi(t)dt$$

$$\int_0^{F_{px,qy}(\mathrm{ht})} \varphi(t)dt \geq \int_0^{\min\{F_{px,qy}(\mathrm{t}), F_{px,px}(\mathrm{t}), \frac{\alpha F_{px,qy}(\mathrm{t}) + \beta F_{qy,px}(\mathrm{t}),}{\alpha+\beta}, F_{px,qy}(\mathrm{t}), F_{qy,qy}(\mathrm{t})\})} \varphi(t)dt$$

$$\int_0^{F_{px,qy}(\mathrm{ht})} \varphi(t)dt \geq \int_0^{\min\{F_{px,qy}(\mathrm{t}), 1, F_{px,qy}(\mathrm{t}), F_{px,qy}(\mathrm{t}), 1\})} \varphi(t)dt$$

$$\int_0^{F_{px,qy}(\mathrm{ht})} \varphi(t)dt \geq \int_0^{F_{px,qy}(\mathrm{t})} \varphi(t)dt$$

Therefore by lemma 4.1, we have px = qy i.e px = fx = qy = gy. Suppose that there is another point z such that pz = fz then by inequality (3.1) we have fz = qy = gy= z, So px = pz and w = px = fx is the unique point of coincidence of p and f . Similarly there is a unique point z ∈ X such that z = qz = gz . Suppose that w ≠ z then by inequality (4.1)

$$\int_0^{F_{px,qy}(\mathrm{ht})} \varphi(t)dt \geq \int_0^{\min\{F_{fx,gy}(\mathrm{t}), F_{fx,px}(\mathrm{t}), \frac{\alpha F_{px,gy}(\mathrm{t}) + \beta F_{qy,fx}(\mathrm{t}),}{\alpha+\beta}, F_{fx,qy}(\mathrm{t}), F_{qy,gy}(\mathrm{t})\})} \varphi(t)dt$$

$$\int_0^{F_{pw,qz}(\mathrm{ht})} \varphi(t)dt \geq \int_0^{\min\{F_{fw,gz}(\mathrm{t}), F_{fw,pw}(\mathrm{t}), \frac{\alpha F_{pw,gz}(\mathrm{t}) + \beta F_{qz,fw}(\mathrm{t}),}{\alpha+\beta}, F_{fw,qz}(\mathrm{t}), F_{qz,qz}(\mathrm{t})\})} \varphi(t)dt$$

$$\int_0^{F_{w,z}(\mathrm{ht})} \varphi(t)dt \geq \int_0^{\min\{F_{w,z}(\mathrm{t}), F_{w,w}(\mathrm{t}), \frac{\alpha F_{w,z}(\mathrm{t}) + \beta F_{z,w}(\mathrm{t}),}{\alpha+\beta}, F_{w,z}(\mathrm{t}), F_{z,z}(\mathrm{t})\})} \varphi(t)dt$$

$$\int_0^{F_{w,z}(\mathrm{ht})} \varphi(t)dt \geq \int_0^{F_{w,z}(\mathrm{t})} \varphi(t)dt$$

Therefore we have w = z then by lemma 4.1, z is a common fixed point of p,q,f and g.

Uniqueness: let z' be the another common fixed point of p,q,f and g then by inequality (3.1)

$$\int_0^{F_{px,qy}(\mathrm{ht})} \varphi(t)dt \geq \int_0^{\min(\{F_{fx,gy}(\mathrm{t}), F_{fx,px}(\mathrm{t}), \frac{\alpha F_{px,gy}(\mathrm{t}) + \beta F_{qy,fx}(\mathrm{t}),}{\alpha+\beta}, F_{fx,qy}(\mathrm{t}), F_{qy,gy}(\mathrm{t})\})} \varphi(t)dt$$

$$\int_0^{F_{pz,qz'}(ht)} \varphi(t)dt \geq \int_0^{\min\{F_{fz,gz'}(t), F_{fz,pz}(t), \frac{\alpha F_{pz,gz'}(t)+\beta F_{qz',fz}(t),}{\alpha+\beta}, F_{fz,qz'}(t), F_{qz',qz'}(t)\})} \varphi(t)dt$$

$$\int_0^{F_{z,z'}(ht)} \varphi(t)dt \geq \int_0^{\min\{F_{z,z'}(t), F_{z,z}(t), \frac{\alpha F_{z,z'}(t)+\beta F_{z',z}(t),}{\alpha+\beta}, F_{z,z'}(t), F_{z',z'}(t)\})} \varphi(t)dt$$

$$\int_0^{F_{z,z'}(ht)} \varphi(t)dt \geq \int_0^{\min\{F_{z,z'}(t), 1, F_{z,z'}(t), F_{z,z'}(t), 1\})} \varphi(t)dt$$

$$\int_0^{F_{z,z'}(ht)} \varphi(t)dt \geq \int_0^{F_{z,z'}(t)} \varphi(t)dt$$

By lemma 4.1, we have z=z'. Therefore z=z' is a unique common fixed point of p,q,f and g.

Theorem (4.2): Let p,q,f and g be a self-maps of Menger space (X, F, T) .Let the pairs (p,f) and (q,g) be occasionally weakly compatible . if there exist h∈ (0,1) such that

$$\int_0^{F_{px,qy}(ht)} \varphi(t)dt \geq \int_0^{\emptyset(\min\{F_{fx,gy}(t), F_{fx,px}(t), \frac{\alpha F_{px,gy}(t)+\beta F_{qy,fx}(t),}{\alpha+\beta}, F_{fx,qy}(t), F_{qy,gy}(t)\})} \varphi(t)dt$$

For all x,y ∈X, and $[0,1]^5 \to [0,1]$ such that $\emptyset(t) > t$ and 0 < t < 1 then there exist a unique common fixed point of p,q,f and g.

Proof: same as theorem (4.1)

Theorem (4.3): Let p,q,f and g be a self-maps of Menger space (X, F, T) .Let the pairs (p,f) and (q,g) be occasionally weakly compatible . if there exist h∈ (0,1) such that

$$\int_0^{F_{px,qy}(ht)} \varphi(t)dt \geq \int_0^{\min\{F_{fx,px}(t), F_{fx,gy}(t), F_{qy,fx}(t), \frac{\alpha F_{px,gy}(t)+\beta F_{qy,fx}(t),}{\alpha+\beta}, F_{px,gy}(t), F_{qx,qy}(t)\})} \varphi(t)dt$$

For all x,y ∈X, and $[0,1]^6 \to [0,1]$ such that $\emptyset(t) > t$ and 0 < t < 1 then there exist a unique common fixed point of p,q,f and g.

Proof: same as theorem (4.1)

5. CONCLUSION

There are many applications of fixed point theory in several field of science.. In this paper the main result is the improved and extended results of fuzzy metric space and Menger space which can be further extended for self-maps with occasionally weakly compatible(owc) conditions and can be used in the finding the solution of LPP, digital problems, economics population conces etc.

6. REFERENCES

1) Abbas M. And Rhoades B. E., Common fixed point theorems for hybrid pairs of occasionally weakly compatible mappings defined on symmetric spaces, Pan American Math.J.18(2008), 35-54.
2) Abbas M. And Rhoades B. E., Common fixed point theorems for hybrid pairs of occasionally weakly compatible mappings satisfying a generalized contractive condition. Math. Comm. 13(2008),pp 295-301.
3) Abbas M. And Rhoades B. E., Common fixed point theorems for hybrid pairs of occasionally weakly compatible mappings satisfying generalized condition of integral type, fixed point theory appl., art. ID 54101, 9 Pages.
4) Fang J.X., Common fixed point theorem and weakly compatible maps in Menger spaces, Nonlinear Anal. TMA 71, (2009), 1833-1843.
5) Hu, X-Q, Common coupled fixed point theorem for contractive mapping in fuzzy metric spaces, Fixed Point theory Appl. 20011, Article ID 363716 (2011).doi:10.1155/2011/363716
6) Jungck G., "Common fixed point for commuting and compatible mappings on compact metric space, Proc. Amer. Math. Soc. 130(1988). 977-983.
7) Jungck G., "Compatible mappings and common fixed points". International J. Math. And Math. Sci., 9(4), (1986), 771-779.
8) Jungct G., Rhoades B.E.,Fixed Point for Occasionally Weakly Compatible mapping, Fixed Point theory, Volume 7, No.2,2006,pp 287-296.
9) Jungct G., Rhoades B.E.,Fixed Point for Occasionally Weakly Compatible mapping, Fixed Point theory, Volume 9, No.1,2008,pp 383-384.
10) Kanan R., "Some results on fixed point". Bull. Cal. Math. (1968) ,60-71.
11) Sedghi,S, Altun, and Shobe.N, Coupled fixed point theorem for contractions in fuzzy metric spaces, Nonlinear Anul.TMA 72(2010),1298-1304. Singh B. and Chauhan M.S., "Generalized Fuzzy metric space and fixed point theorems", Bull. Cal. Math. Soc.89, 105-112(1997).
12) Singh B., Chauhan M.S ., Common fixed points of maps in fuzzy metric spaces, International Journal of Math. Math. Sci. 17(1999), pp 253-258.
13) Singh B., Jain S., A fixed point Theorem in Menger space through weak compatibility, J.Math. Anal. Appl.301(2005) 439-448.
14) Singh. B and Chauhan M.S., "Common fixed points of compatible maps in fuzzy metric spaces", Fuzzy Sets and Systems 115(2000), 471-475

15) Patel Jatin S. "Some fixed point theorems for Occasionally Weakly compatible Mapping in fuzzy 2-metric spaces",ww.ijrst.org vol-1 (2016),pp 01-85
16) Mishra R.K and Choudhary Sanjay,''On fixed point theorem in fuzzy metric spaces',IMACST ,vol-1,No.1(2010),pp 45-47
17) Thagafi A.A and Shahzad N.,"Generalized 1-nonexpansive selfmaps and Invariant Approximation' Acta Mathematica Sinica,English series May,2008,Vol-24,No.5,pp.867876.
Schweizer B. and Sklar A. , Probabilistic Metric space,North-Holland Series in Probability and Appl. MathNorth-Hollan Publishing Co., New York (1983).
18) Menger K., Statistical Metrices, Proceedings of the National Academy of sciences of the (USA).Vol 28 (1942) ,535-537
19) Pant R.P, A Common fixed point theorem under a new condition .Indian Journal of Pure and Applied Mathematics.vol 30 (1999),147-152
20) Rohan Y.,Singh M.R.,Shambu L., Common fixed points of compatible mapping of type (C) in Banach space .Proc.Math.Soc. Vol 20,(2004),77-87.
21) Singh M.R, Singh Y.M., Compatible mapping of type (E) and common fixed point theorems of Meirkeeler type . Int.J.Sci.Eng. Appl. Vol 19 (2007), 299-315.
22) Singh Ruchi et al.,Fixed point Results in fuzzy Menger space with common property(E.A). Journal of Engineering Research and applications vol-5,Issue 8 (Aug-2015),pp-62-67.